LIMIT STATES ANALYSIS OF BRIDGE ABUTMENTS BY COMPUTER-AIDED METHOD

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ABSTRACT

This paper presents a computer program developed for limit states analysis of abutments. The program can perform both structural and geotechnical analysis of bridge abutments and check their resistances in compliance with limit states design criteria. In the program, the earth pressure coefficient for the backfill soil is calculated as a function of abutment's lateral non- linear displacement. Therefore, for abutments partially restrained against lateral movement, an earth pressure coefficient less than that of at-rest conditions may be obtained. This may result in a more economical design.

1. Introduction

Limit states are conditions under which a structure can no longer perform its intended functions. The limit states design (LSD) process considers two conditions to satisfy; the ultimate and the serviceability limit states. The ultimate limit states (ULS) are related to the safety of the structure and they define the limits for its total or partial collapse. The serviceability limit states (SLS) represent those conditions, which adversely affect the expected performance of the structure under service loads.

LSD has received particular attention in the geotechnical and structural engineering literature over the last three decades. Many researchers and practicing engineers have documented their findings on this subject. Guidance with the application of limit state design procedures is available through a number of design codes. However, the application of LSD to substructures is more recent. In the past, substructure design was based on allowable stress or working stress design (WSD), while the superstructure design was based on LSD. The use of LSD philosophy for superstructures and WSD philosophy for substructures led to confusions. The confusion potentially exists with respect to loading at soil-structure interface for the evaluation of ultimate limit states (ULS). The structural engineer employing the LSD approach for the design of substructures thinks in terms of factored loads to be supported by the bearing soil. The geotechnical engineer using WSD approach in soil bearing capacity assessment thinks in terms of nominal loads and allowable soil pressures. Therefore, the geotechnical report provides the structural engineer with the values of allowable soil pressure. The structural engineer then interprets the meaning of the recommended soil pressure and factors it in an effort to compare it with the responses due to factored structural loads. Nevertheless, the recommended soil bearing pressure may be controlled by settlement considerations or SLS rather than bearing failure considerations or ULS. Obviously, a sense for the actual level of safety has been lost through the incompatible design process. The foundation may be over designed resulting in loss of economy rather than the improved economy that LSD is supposed to provide. Therefore, it became evident that a limit state approach was required for geotechnical design.

The LSD process for the design of bridge abutments is more tedious than WSD process. It requires two different analyses to satisfy the structure performance for both SLS and ULS. The ULS itself requires more than one analysis to satisfy the geotechnical and the structural limit states. Generally, designers try to obtain the optimal structure dimensions to satisfy the limit states criteria by following a trial -and-error analysis and design procedure. Nevertheless, a manually performed trial-and-error analysis and design iteration to obtain the optimal structure dimensions and reinforcement under different loading conditions and given limit states criteria could be inaccurate, tedious and time consuming. Considering these, a computer Program, ABA, for the analysis and design of bridge abutments has been developed.

In the subsequent sections, first, the general features of the program, ABA are described. This is followed by a brief description of the general program structure. Next, the procedure used in the program for calculating backfill pressure coefficient is defined. Then, the LSD procedure implemented in the program for bridge abutment design is introduced. This included the procedure for checking the stability of the structure for sliding and overturning and the calculation of base pressure, pile axial forces, structural responses and resistances. Following this, simple design-aid charts for retaining walls are introduced.

2. General features of the program

ABA is capable of analyzing bridge abutments and retaining walls and checking their structural and geotechnical resistance using LSD criteria. Retaining walls are programmed as a sub-element of abutments and therefore the word abutment will mean both retaining wall and abutment thereafter.

2.1 Abutment Geometry

ABA analyses the general type of reinforced concrete abutment. The generic shape of the abutment's wing-wall is defined in the program; the geometry of the abutment can be modified by assigning constraints to its dimensions. The ballast-wall and the breast-wall parts of the abutment obtained by assigning various constraints to their dimensions.

The geometry and local coordinate axes of the abutment footing for abutments with deep foundations, piles are defined in rows extending along footing local coordinate axes. The number of piles in a row and the location of each row from the centroid of the footing are input by the user. Each row may have piles with identical batters perpendicular to row direction. If a row contains piles with different batters, then it may be defined as a combination of two or more rows of piles located at the same distance from the centroid of the footing. The piles at both ends of each row may also have batters parallel to row direction. The piles are assumed to have constant spacing within a row and located symmetrically with respect to footing local coordinate axis.

2.2 Loads

The types of loads allowed by the program to act on the abutments are; concentrated loads at the bearings, surcharge pressure, backfill soil pressure, soil compaction load, self weight of the abutment, backfill soil and barrier walls on wing-walls. The concentrated loads may belong to one of permanent, transitory or exceptional load groups. Surcharge pressure is assumed to act over the entire surface area of the backfill soil at the abutment top level. It may belong to either permanent or transitory load group. The backfill soil pressure is either calculated internally by the program as a function of structure lateral displacement or it can be defined externally by the user. The user defines compaction load at the surface of the backfill soil. The program then internally defines the linearly varying lateral earth pressure due to this compaction load. The program also internally calculates the self-weight of the abutment, backfill soil, and barrier-walls-on-wing-walls.

3. General program structure

ABA consists of a control program that manages the database, an analysis and design engine and a graphical user interface (GUI) for user data input. The analysis and design engine consists of three modules; abutment analysis module, footing analysis module and resistance module. The abutment analysis module performs the analysis of the abutments excluding the footing part. The footing analysis module then performs the analysis for the footing part of the abutment or it can be operated independently. The resistance module then calculates the structural resistance of any specified cross-section on the structure. the control program first allows the user to define the properties of the abutment and its footing using the GUI. The user-defined data is then stored in a structured database, which contains material, geometry, loading data and control flags. The program then uses this database for the analysis and design of the abutment. The control program operates the necessary sub-module depending on the analysis type. If footing is analyzed as part of abutment, then the control program first initiates the abutment analysis module. Next, it stores the footing load data, obtained from the abutment analysis module, in the database. Then, it calls the footing analysis module to complete the analysis. Finally, it calls the resistance module to perform structural resistance calculations.

4. Calculation of earth pressures

In the program, separate earth pressure conditions are considered behind the abutment for geotechnical and structural LSD. For the geotechnical LSD, active earth pressure condition is considered behind the abutment as the structure is assumed to rotate at its base or displace away from the backfill at the verge of geotechnical limit state conditions such as overturning or sliding of the structure. Such movements will mobilize the soil to an active state of equilibrium. For structural LSD, the structure is assumed to have no such movements. Accordingly, an earth
pressure ranging between active and at-rest conditions is considered for the structural design of the abutment The actual earth pressure coefficient, K, may change between active, K_a and at-rest, K_o, earth pressure coefficients depending on the amount of lateral deformation of the abutment due to the permanently applied loads. Past researchers obtained the variation of earth pressure coefficient as a function of structure top displacement from experimental data and finite element analyses. For practical purposes, this variation is assumed as linear.
This linear relationship is expressed as:

$$K = K_o - \phi\, d \geq K_a$$

Where, d is the top displacement of the earth retaining structure away from the backfill soil and ν, is the slope of the earth pressure variation depicted. The calculated top displacement of the abutment and the active and at-rest earth pressure coefficients are substituted into the above equation to obtain the actual earth pressure coefficient for the structural design. A similar approach was followed elsewhere to estimate the passive earth pressure coefficient for the backfill soil for the design of integral-abutment bridges. In the program, Coulomb theory is used to calculate the active and at-rest lateral earth pressure coefficients assuming zero friction between the wall surface and the backfill. The effect of backfill slope on the active earth pressure coefficient is also considered in the program.

5. Structure Model

The structure in the program for the calculation of abutment top displacement. Only the effects of unfactored dead loads and backfill pressure are considered in the calculations. The eccentricities due to the dead load reactions on the bearings are also taken into consideration by applying a concentrated moment at the point of application of the dead loads on the structure model. The abutment is modeled as a cantilever having a unit width and a variable cross-section along its height. The cantilever element is then connected to the footing member. The footing is modeled as a vertical rigid bar with a rotational spring connected to its end. The length of this rigid bar is set equal to the footing depth, h_f. The rotational spring at the end of the rigid bar simulates the effect of footing rotation on the magnitude of abutment top displacement. The loads acting on the abutment are proportioned to the unit width of the abutment.

The bridge deck may restrain the lateral displacement of the abutment. The degree of this restraint is based on the type of bearings used. For frictional bearings, the restraining force is equal to the total dead load reaction force on the bearing, multiplied by the coefficient of friction for the type of bearing used. In the program, first a fictitious rigid lateral support is introduced in the structure model at the bearing location. Next, the lateral reaction force due to the applied loads is calculated at this support. If the restraining force provided by the bearings is smaller than this reaction force Otherwise, the movement is assumed to be totally restrained and the earth pressure coefficient is set equal to K_o. For elastomeric bearings, the restraining force is proportional to the lateral displacement of the abutment at bearing's location and the stiffness of the bearing. A spring with stiffness identical to that of the elastomeric bearings is placed at the bearing location to simulate the restraining effect of the bearings. The stiffness of this spring per unit width of abutment is expressed in the program as:

$$K_b = \frac{n_b G_b A_b}{h_b w_a}$$

Where, n_b is the number of bearings, G_b is the shear modulus of the bearing material, A_b is the plan area of the bearing, h_b is the bearing height and w_a is the total width of the abutment.

For bearings providing lateral fixity at the abutments, the movement is assumed to be restrained and the earth pressure coefficient is set equal to K_o. In the case of cantilever retaining walls, no restraint is considered in the displacement calculation.

The stiffness of the rotational spring in the model is determined by the rotational stiffness of the footing. For shallow foundations, the rotational stiffness, $K_{\theta f}$, of the footing is expressed as

$$K_{\theta f} = \frac{1}{12} B_1^3 B_2 k_s$$

Where, B_1 and B_2 are the plan dimensions of the footing respectively parallel and perpendicular to the bridge longitudinal directions and k_s is the coefficient of sub-grade reaction for the bearing soil input by the user. In the case of pile foundations, the rotational stiffness of the foundation is calculated in the program as:

$$K_{\theta f} = \sum_{i=1}^{nr} n_i \frac{E_p A_p}{L_p} d_i^2$$

Where, nr is the number of pile rows, E_p is the modulus of elasticity of pile material, A_p and L_p are respectively, the cross -sectional area and length of a single pile and d_i is the distance of pile row i, from the geometric centerline of the footing.

A closed form solution for the reaction forces at the translational and rotational springs in the model is obtained for each type of load applied on the structure and implemented in the program.

Calculation of Top Displacement

The displacement, δ^{*}_{T}, at the top of the abutment is calculated in the program using the following equation.

$$\delta_T = \frac{M_b (h_a + h_f)}{K_{\theta f}} + \int_0^{h_a} m \frac{M}{E_c I_a} dx$$

Where, M_b is the total moment at the footing base, M and m are moments, respectively, due to the externally applied loads and a horizontal unit dummy load applied at the top of the abutment, E_c is the modulus of elasticity of abutment concrete and I_a and h_a are respectively the moment of inertia and height of the abutment. The first set of terms in the above equation represents the contribution of footing rotation to the top displacement. The integration represents the contribution of the abutment's flexural deformation to the top displacement and is obtained using the unit dummy load method. Note that the expression, M/E_cI_a, in the integral is the curvature of the abutment due to the applied loads. The integration is performed numerically using the trapezoidal rule of numerical integration method. The structure model is first divided into 100 segments and the resulting segment length is used as an integration step. The moment, M, due to externally applied loads is then calculated at each point of integration. Next, the inelastic curvature ($\varphi = M/E_cI_a$) corresponding to the applied moment and axial force is calculated using nonlinear material models for concrete and steel to obtain an accurate estimate of structure displacement. The procedure followed to calculate the curvature is defined in the subsequent sections. The moment, m, due to the unit dummy load is also calculated at the integration points and multiplied by the calculated curvature and an integration factor which is a function of the type of numerical integration method used. For this particular case, the integration factor is 0.5 for the first and last points of integration and 1.0 for the rest. Finally, the top displacement due to the flexural deformation of the abutment is obtained by summing up the results obtained for each integration point and multiplying the sum by the integration step.

6. Structural Analysis of Footing

For the structural analysis of the footing, two different ULS soil pressure distributions are considered in the program. The first case considers a contact pressure distribution due to yielding soil, which approximates a uniform pressure distribution over an effective area, as explained previously. This pressure distribution is primarily used to check the bearing resistance of the soil. However, the abutment footing is also structurally designed to sustain such a pressure. The second case assumes a nearly rigid footing and a linear contact pressure distribution due to an elastic non-yielding soil where the probable resistance of soil may exceed the ultimate resistance used in geotechnical design. The program then calculates the flexural and shear forces in the footing for each contact pressure distribution. Larger of the structural responses obtained from both cases will then govern the structural design at ULS. For the SLS condition, only a linear contact pressure distribution is assumed. In the case of deep foundations, flexural and shear forces in the footing are calculated using the previously calculated SLS and ULS pile axial forces.

Normally, the program calculates flexural forces at both faces of the abutment wall and shear forces at a distance 0.9 times the footing thickness from both faces of the abutment wall. Additional sections can be specified by the user around pile locations in the case of deep foundations. The calculated flexural and shear forces are then divided by the width of the footing to obtain the effect of such forces per unit width. The structural resistance calculations are then performed at the same response locations by the program's resistance module.

7. Structural Resistance Calculations

The optimum flexural resistance of a reinforced concrete section is a function of the applied axial force and the extreme fiber compression strain. To calculate the flexural resistance of a cross section along the structure for a prescribed axial force, the extreme fiber compression strain $_{cu}$, for concrete is varied between 0.0020 and 0.0035 using an incremental step of 0.0001. For each incremental strain value, the slope of the strain diagram is established for an assumed location, *c*, of neutral axis measured from the top of the section as shown in Figure 8. Corresponding compressive and tensile stresses in concrete and steel are determined from material models described previously. Internal forces in concrete, as well as reinforcing steel are calculated. The equilibrium is checked by comparing the resultant internal force with the externally applied axial force. If the equilibrium is satisfied within a prescribed range of accuracy, the assumption for neutral axis location is verified. Otherwise, the neutral axis location is revised and the same process is repeated until the equilibrium is satisfied. Next, the internal moment is calculated and stored in an array. The program then continues the analysis with the next selected extreme compression fiber strain until it reaches the maximum value of 0.0035. At the end of the analyses, the maximum of the stored moments is selected as the flexural resistance of the section.

The compression field theory is implemented in the program to calculate shear resistance of a cross section on the structure. The shear resistance, V_r, of a reinforced concrete section without transverse reinforcement is defined as:

$$V_r = \beta\varphi_c f_{cr} b_v d_v$$

where, β is a dimensionless parameter, φ_c is the resistance factor for concrete, b_v and d_v are respectively the effective section width and depth used in shear resistance calculations. To calculate β , the angle of inclination, θ, of principle compressive strain or shear cracks is varied between 27^o and 79^o using an incremental step of 1^o in the program. For each incremental value of θ , the reinforcement tensile strain, ε_x, is calculated using the following equation:

$$\varepsilon_x = \frac{0.5\left(P_f + V_f Cot\theta\right) + \frac{M_f}{d_v}}{E_s A_s} \geq 0$$

Where, P_f, V_f and M_f are respectively the factored axial load, shear and moment acting on the cross section and E_s and A_s are respectively the modulus of elasticity and area of reinforcing steel. Then, the principal tensile strain, ε_1, and β are calculated as:

$$\varepsilon_1 = \varepsilon_x \left(1 + \cot^2\theta\right.$$

$$\beta = \frac{0.36}{0.3 + \frac{0.69d\varepsilon_1}{}} \leq \frac{0.66\cot\theta}{1 + \sqrt{500\varepsilon_1}}$$

Where, d is the distance of tensile reinforcement from the extreme compression fibre. The value of β is stored in an array and the procedure is repeated for the next incremental value of θ until it reaches the maximum value of 79^o. At the end of the analysis, the maximum of the stored β values is used to calculate the shear resistance of the section

Conclusions

A computer program, developed for the limit states analysis of bridge abutments, is presented in this paper. Although several other computer programs exist for the analysis of bridge abutments, they are limited to cases where working stress design approach is used for structural analysis. Different from these conventional programs, the developed program is able to perform structural analysis of bridge abutments and check their resistance to calculated responses using limit states design criteria. In the program, the earth pressure coefficient for the backfill soil is calculated as a function of abutment's lateral displacement taking into consideration the non-linear force-deformation relationship of the structure. Therefore, for abutments partially restrained against lateral movement, an earth pressure coefficient less than that of at-rest conditions may be obtained. This may result in a more economical design. Design-aid charts for cantilever retaining walls are also generated using this program.

REFERENCES

1. Meyerhof, G.G. Safety factors in soil mechanics. Canadian Geotechnical Journal 1970; 7: 349-355.
2. Meyerhof, G.G. Limit states design in geotechnical engineering. Structural Safety 1982; 1: 67-71.
3. Meyerhof, G.G. Safety Factors and limit states analysis in geotechnical engineering. Canadian Geotechnical Journal 1984; 21: 1-7
4. Meyerhof, G.G. Development of geotechnical limit state design. Proceedings of the International Symposium on Limit State Design In Geotechnical Engineering . Copenhagen: Danish Geotechnical Society, 1993; 1: 1-12.
5. Meyerhof, G.G. Development of geotechnical limit state design. Canadian Geotechnical Journal 1995; 32: 128-136.
6. Lumb, P. Safety factors and probability distribution of soil strength. Canadian Geotechnical Journal 1970; 7: 225-242.
7. Allen, D. E. Limit States Design - A probabilistic study. Canadian Journal of Civil Engineering 1975; 2: 36-49.
8. Allen, D. E. Limit states criteria for structural evaluation of existing buildings. Canadian Journal of Civil Engineering 1991; 18: 995-1004.
9. MacGregor, J.G. Safety and limit states design for reinforced concrete. Canadian Journal of Civil Engineering 1976; 3: 484-513.
10. Bolton, M. D. Limit state design in geotechnical engineering. Ground Engineering 1981; 14(6): 39-46.
11. Balkie, L. D. Total and partial factors of safety in geotechnical engineering. Canadian Geotechnical Journal 1985; 22: 477-482.
12. Ovesen, N.K. Towards an european code for foundation engineering. Ground Engineering 1981; 14 (7): 25-28.
13. Ovesen, N.K. Eurocode 7: An european code of practice for geotechnical design. Proceedings of the International Symposium on Limit State Design In Geotechnical Engineering. Copenhagen: Danish Geotechnical Society, 1993; 3: 691-710.
14. Ovesen, N. K. and Orr, T. Limit States Design: the european perspective. Proceedings of

Geotechnical Engineering Congress 1991. American Society of Civil Engineers, 1991; Special Publication No 27, 2: 1341-1352.

15. Green, R. The development of a LRFD code for Ontario bridge foundations. Proceedings of Geotechnical Engineering Congress 1991. American Society of Civil Engineers, 1991; Special Publication No 27, 2, 1365-1376.

16. Green, R., (1993), 'LSD Code for Bridge Foundations', Proceedings of the International Symposium on Limit State Design in Geotechnical Engineering. Copenhagen: Danish Geotechnical Society, 1993; 2, 459-468.

17. Barker, R. M., Duncan, J. M. K., Rojiani, K. B., Ooi, P. S. K, Kim, S.G. Manuals for the design of bridge foundations. NCHRP Report 343. Washington D.C.: Transportation Research Board; National Research Council, 1991.

18. Becker, D. E. Eighteenth Canadian Geotechnical Colloquium: Limit States Design for foundations. Part I. An overview of the foundation design process., Canadian Geotechnical Journal 1996; 33, 956-983.

19. Ontario Highway Bridge Design Code. Third Edition, Ministry of Transportation, Quality and Standards Division, Downsview, Ontario, Canada, 1991.
20. European Code for Standardization. Eurocode 7: Geotechnical design, general rules. Danish Geotechnical Institute, Copenhagen, Denmark, 1992.
21. Associate Committee on the National Building Code. National building code of Canada. National Research Council, Ottawa, Canada, 1995.
22. Canadian Highway Bridge Design Code - Final Draft. Canadian Standards Association, Toronto, Ontario, Canada, 2000.
23. Clough, G. M., Duncan, J. M. Foundation engineering handbook. Fang, H.Y. editor. New York: Van Nostrand Reinhold, 1991.
24. Dicleli, M. A rational design approach for prestressed-concrete-girder integral bridges. Engineering Structures 2000; 22(3): 230-245.
25. Dicleli, M. A simplified structure model for computer-aided analysis of integral-abutment bridges. ASCE Journal of Bridge Engineering 2000; 5(3): 1-9
26. Demetrios, E. T. Bridge engineering: design, rehabilitation and maintenance of modern highway bridges. New York: McGraw-Hill, 1995.
27. Priestly, M. J. N., Seible, F., Calvi, G. M. Seismic design and retrofit of bridges. New York: John Wiley and Sons, 1996.
28. Ghali, A and Neville, A. M. Structural analysis: a unified classical and matrix approach, 3rd edition. New York: Chapman and Hall, 1989
29. Maron, M. J. Numerical analysis: a practical approach, 2nd edition. New York: Macmillan, 1987.

30. Saatcioglu, M. and Razvi, S. Strength and ductility of confined concrete. ASCE Journal of Structural Engineering 1992; 118(9): 2421-2438.
31. Duan, L., (1996), Bridge-column footings: an improved design procedure. ASCE Practice Periodical on Structural Design and Construction 1996; 1(1): 20-24.