AN EXTENDED COMPANDING ALGORITHM FOR PAPR REDUCTION IN OFDM

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Abstract: *PAPR is a challenging task in Orthogonal Frequency Division Multiplexing(OFDM) systems, which influence on OFDM system performance. Several methods are proposed to reduced the PAPR. Among all these companding transform are widely used. In this paper an extended companding method is proposed and compared with existing methods. This method is reducing the PAPR and also improve the Bit Error Rate and total system performances*

1. INTRODUCTION

The concept of OFDM is quite simple but the practically of implementing it has many complexities. OFDM is a digital multicarrier modulation scheme uses a large number of closely spaced orthogonal sub carriers. OFDM system allows the carrier, which are orthogonal to each other meaning that cross talk between co-channels is eliminated and inter carrier guard bands are not required.

OFDM is a digital multicarrier modulation scheme, which uses a large number of closely spaced orthogonal sub-carriers [3]. A single stream of data is split into parallel streams each of which is coded and modulated on to a sub carrier, a terms commonly used on OFDM system.

OFDM is special form of Multi Carrier Modulation (MCM) with densely spaced sub carrier with over lamping spectra ,Thus allowing for multiple-access.MCM is the principle of transmitting data by dividing the stream into several bit streams, each of which has a much lower bit rate, and by these sub-streams to a modulated several carriers. This technique is being investigated as the next generation transmitted scheme for mobile wireless communication.

The High peak –to-average power ratio (PAPR) is one of the major problem in OFDM system. Which gives effects on analog to digital (A/D) and digital to analog (D/A) converts complexity in OFDM system.

This PAPR is also reduces the efficiency of power amplifies those used in OFDM systems. And also the synchronization error can destroy the orthogonality and cause interference. A lot of effort is required to design accurate frequency synchronizers for OFDM . Various techniques are used to reduce the PAPR, such as block code, selective mapping (SLM) ,one tone reservation and active constellation extension(ACE) etc..

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There are different companding techniques are used to reduced the effect of PAPR, by simplest method of clipping operation. Clipping is a highly nonlinear process, however. It generates significant out-of-band interference (OBI) [2]. The method was first proposed based on μ-law algorithm, which gives better performance than clipping method. Since there was different companding transform methods with better result have been published.

This paper is organized as follows. In the II section the PAPR problem in OFDM is overviewed. Section III presents the proposed algorithm and its theoretical analysis. Section IV describes the simulation results. Conclusions are given in section V.

II. PAPR PROBLEM AND PAPR REDUCTION

As explained earlier, one of the major drawbacks of OFDM is the very high peak-to-average power ratio (PAPR). PAPR of OFDM increases exponentially with the number of subcarriers. If power amplifiers are not operated with large power back-offs, it is impossible to keep the out-of- power below the specified limits. This situation leads to very inefficient amplification and expensive transmitters, so it is highly desirable to reduce the PAPR. In this section a detailed mathematical analysis for PAPR is presented.

For one OFDM symbol with N subcarrier, the complex baseband signal can be written as

$$\chi(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n)\, e^{\frac{j2\pi nt}{N}} \quad ,0 \le t \le T \qquad (1)$$

Where X(n) = [$X(0), X(1), \ldots, X(N-1)$], N is the number of subcarriers and T is the duration of the OFDM symbol. For large N, the real and imaginary values of $\chi(t)$ become Gaussian distributed, each with a mean of zero and a variance ½. The amplitude of the OFDM signal therefore has a Rayleigh distribution, while the power distribution becomes a central chisquare distribution given by $F(z) = 1 - e^{-z}$ [10]. Therefore here it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude. So the peak amplitude of OFDM signal must be limited.

PAPR is mathematically defined as:

$$\text{PAPR} = 10\log_{10}\frac{\max[\,|\chi(t)|^2]}{\frac{1}{T}\int_0^T|\chi(t)|^2 dt}\ (\text{dB}) \quad (2)$$

It is easy to say from (2), by decreasing the numerator $\max[|\chi(t)|^2]$, increasing the denominator $(1/T)\int_0^T|x(t)|^2 dt$, or both, the PAPR reduction may be achieved. The effectiveness of a PAPR reduction technique is measured by the complementary cumulative distribution function (CCDF), which is the probability that PAPR exceeds some threshold i.e.. $p0$.

CCDF = Probability (PAPR > $p0$), (3)

III.NEWCOMPENDING ALGORITHAM

OBI is caused by the spectral leakage into alien channels, and also caused by companding, which requires the knowledge of the power spectral density (PSD) of the commanded signal. But analytical expression of the PSD is in general mathematically difficult, because of the nonlinear companding transform involved. Therefore it is proposed an alternative solution to analyse the OBI. Let $f(x)$ be a nonlinear companding function, an $\chi(t) = \sin(\omega t)$ be the input to the compander. The commanded signal $y(t)$ is:

$$y(t) = f[x(t)] = f[\sin(\omega t)]\,. \quad (4)$$

Since $y(t)$ is a periodic function with the same period as $\chi(t)$, $y(t)$ can then be expanded into the following Fourier series:

$$y(t) = \sum_{k=-\infty}^{+\infty} c(k)e^{jkwt}\,, \quad (5)$$

where the coefficients $c(k)$ is calculated as:

$$c(k) = (-k) = \frac{1}{T}\int_0^T y(t)e^{-jkwt}\,dt \quad ,T=\frac{2\pi}{\omega}\,. \quad (6)$$

Here the input *x(t)* in this case is a pure sinusoidal signal. Any $c(k) \neq 0$ for $k > 1$ is the OBI produced by the nonlinear companding process. Hence, to minimize the OBI, $c(k)$ must be zero fast enough as k increases. It has been shown that $c(k)\cdot k^{-(m+1)}$ tends to zero if $y(t)$ and its derivative up to the m-th order are continuous [8]. Given an arbitrary number *n*, the n-th order derivative of $y(t)$, $d^n y/dt^n$ is a function of $d^i(x)/dx^i$, (i = 1, 2, $\cdots$, n), as well as $\sin(\omega t)$ and $\cos(\omega t)$, i.e.:

$$\frac{d^n y}{dt^n} = g[\frac{d^n y}{dx^n}, \frac{d^{n-1}y}{dx^{n-1}}, \cdot\cdot, \frac{df(x)}{dx}, \sin(wt), \cos(wt)]. \quad (7)$$

$\sin(\omega t)$ and $\cos(\omega t)$ are continuous functions, $d^n y/dt^n$ is continuous if and only if $d^i f(x)/dx^i$, (i = 1, 2, $\cdots$, n) are continuous. Therefore we can conclude:

Companding gives minimum value of OBI if the companding function f(x) is infinitely differentiable.

The functions that meet the above condition are the smooth functions.

The proposed an extended companding algorithm using a Smooth function, namely the airy special function. which is given as follows:

$$f(x) = \beta\cdot\text{sign}(x)\cdot[\text{airy}(0) - \text{airy}(\alpha\cdot|x|]\,, \quad (8)$$

Where airy (·) is the airy function of the first kind. α is the parameter that manages the degree of companding that means PAPR. β is the factor adjusting the average output power of the compander to the same level as the average input power:

$$\beta = \sqrt{\frac{E[|x|^2]}{E[|airy\ (0) - airy\ (\alpha.|x|)|^2}} \quad (9)$$

where $E[\cdot]$ denotes the expectation.
The decompanding function is the inverse of $f(x)$:

$$f^{-1}(x) = \frac{1}{\alpha}.sign(x).airy^{-1}[airy(0) - \frac{|x|}{\beta}] \quad (10)$$

Here the superscript -1 gives the inverse operation and observe that the input to the decompander is a quantized signal with finite set of values. Therefore numerically pre-compute $f^{-1}(x)$ and use table look-up to perform the decompanding in practice.

Now to evaluate the BER performance of this proposed algorithm. Assume $y(t)$ denote the output signal of the compander, $w(t)$ the white Gaussian noise. The received signal can be expressed as:

$$z(t) = y(t) + w(t). \quad (11)$$

The decompounded signal $\tilde{\chi}(t)$ simply is:

$$\tilde{x}(t) = f^{-1}[z(t)] = f^{-1}[y(t) + w(t)] \quad (12)$$

Observe that the signal-to-noise ratio (SNR) in a typical additive white Gaussian noise (AWGN) channel is much greater than 1.

Using the first order Taylor series expansion, (12) can be Approximated as follows:

$$\tilde{x} \approx x(t) + \frac{df^{-1}(u)}{du}/_{u=y(t)}.\,\omega(t) \quad (13)$$

Equation (13) represents the range of the decompanding function $f^{-1}(u)$ where $df^{-1}(u)/_{u=y(t)} < 1$, the noise $w(t)$ is suppressed, and if $y(t)$ is out of the range, $df^{-1}(u)/_{u=y(t)} > 1$ and the noise is enhanced. Therefore, if the parameter α in (8) is properly chosen such that more $y(t)$ is within the noise-suppression range of $f^{-1}(u)$, it is possible to achieve better overall BER performance. It is worth to mention though that BER and PAPR affect each other adversely and therefore there is a trade off to make.

IV.SIMULATION RESULTS

In this simulation the OFDM system used 64 QPSK-modulated data points. The size of the FFT/IFFT is 256, meaning a 4× oversampling. Here the compander input power of 3dBm, the parameter α in the companding function is chosen to be 40.

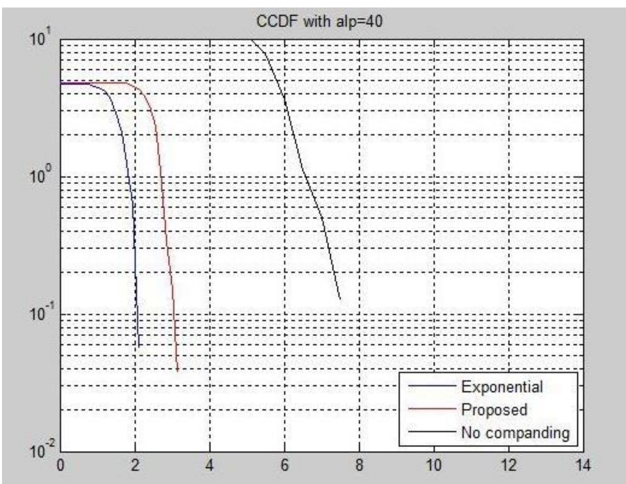

Fig:1. Complementary cumulative distribution function of original and companded signals (compander input power =30dBm, α=40)

Consequently about percent of $y(t)$ is within the noise-suppression range of the decompanding function. The other popular companding algorithm, namely the exponential companding algorithm [4], is also included in the simulation for the purpose of performance comparison.

The simulated PSD of the companded signals are also analysed. The proposed algorithm produces OBI almost 3dB lower than the exponential algorithm, 10dB lower than the μ-law algorithm. Fig. 1 depicts the CCDF of the two companding schemes.

The new algorithm is roughly 3.8 dB superior to the exponential. Our algorithm outperforms the other one. The amount of improvement increases as SNR becomes higher. One more observation from the simulation is: unlike the exponential companding whose performance is found almost unchanged under different degrees of companding, the proposed algorithm is flexible in adjusting its specifications simply by changing the value of α in the companding function.

V. CONCLUSION

In this paper, it is proposed an extended companding algorithm. Both theoretical analysis and computer simulation show that the algorithm offers improved performance in terms of BER and reducing PAPR effectively.

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