EFFECT OF ELASTIC MODULI OF BRITTLE MATRIX COMPOSITES DUE TO INTERFACIAL DEBONDING

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Abstract: *Elastic moduli of brittle matrix composites with uni-symmetric and doubly-symmetric interfacial debonding are presented in this paper. Traction continuity and displacement continuity conditions are imposed along the boundary of adjacent representative elements. The representative volume element boundaries do not remain straight for the composite under loading. Parametric studies assessing the effect of the debonding angle, the shear moduli ratios in the constituents and the fiber volume fractions on the composite shear moduli are also presented*

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1. INTRODUCTION

The mechanical properties of fiber reinforced composites can be significantly affected by bond between the fibers and the matrix. In brittle matrix composites the constituents are both brittle in nature and the fibers are weakly bonded to the matrix. Although this weak bond is detrimental to compressive and transverse strength properties, it is believed to be an important source of enhancing strength and fracture toughness in these composite systems. Recent experimental studies on the BMCs have also shown that the degree of bonding between the fiber and the matrix can dominate their mechanical properties and associated failure modes. In order to achieve optional performance between strength and stiffness for composite development, the effect of weak bond or debonded interface conditions on the mechanical properties of composite materials needs to be fully understood.

The prediction of effective mechanical properties of composite materials can be approached in many ways. The viable approach for engineering applications is based on a theory which replaces the actual homogeneous medium by an equivalent anisotropic homogeneous continuum if the scale of the deformation is sufficiently larger than the characteristic length of the microstructure. By further assuming the periodicity of the microstructure, the effective elastic module of the composites are determined by the elastic properties of the constituents and internal geometry of the representative volume element (RVE). Many analytical and numerical studies have been carried out on the determination of the elastic properties of composite materials with perfectly bonded interface. Usually, concentric cylinders, square array, hexagonal array are assumed for mathematical models. Hashin and Rosen(1964) provided the lower and upper bounds of elastic moduli based on variational principles.

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Semi analytical approaches have been treated by chin and Cheng(1971) to evaluate elastic moduli of the composite. Sun and Vidya (1993) predicted elastic moduli of composite by employing periodic displacement conditions along the cell boundaries and using an energy equivalence consideration with a three dimensional finite element analysis.

The effect of weak bond or debonded interface on the mechanical properties has been studied by several investigations. Pagano and Tandon (1990) developed an approximate model by assuming various interfacial conditions. Several definitions of composite strain were used in the determination of effective moduli. Shimansky etal used a finite element method to predict transverse moduli of the debonded interface in a ceramic matrix composite.

In this paper, an analysis using finite element methods has been applied to fiber reinforced brittle matrix composites in order to predict the influence of the debonded interface on effective elastic moduli of the composites. The extent of the debonded interface is simulated by uni-symmetric and doubly-symmetric debonding geometries. The debonded interface is assumed to be completely separated. The shear moduli ratios is the constituents and fiber volume fraction on the composite shear modului are also studied. The geometrical layout of the composition with periodic rectangular array model is considered in the analysis. The traction continuity and displacement continuity constraint equations are imposed for different deformation modes in order to maintain the geometric compatibility and static equilibrium between neighboring RVE.

A two phase composite with repeating geometry is considered. when the unidirectional reinforced composite is subjected to a uniform macroscopic applied stress, the stress distribution will depend on the properties of the constituents, interfacial conditions and the geometry of the representative volume element. A rectangular RVE with dimensions shown in fig.1. Two different cases of debonding are considered. In uni-symmetric debonding, the fiber is perfectly bonded to the matrix expect over the interface region and in doubly-symmetric debonding, the interface region and The axis coincides with the fiber axis. Under deformation, each RVE in the composite must experience identical displacement and stress fields. Therefore the constraints must be imposed on the boundary of each cell so that the displacements and stresses are compatible with the displacements and stresses on the neighboring cells.

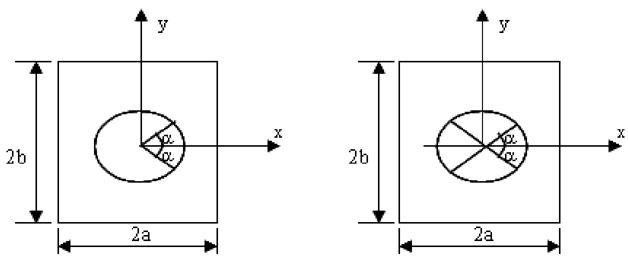

Fig.1. Interfacial debonding of a representative volume element.

Young's moduli and poison's ratios:

To calculate the Young's moduli and poison's ratios of the composite, a periodic rectangular array under generalized plane strain but excluding shear loadings is considered. The displacement compatibility conditions on the boundary of the RVE are.

$$\begin{aligned} u(a,y) &= u(-a,y) + 2a\varepsilon_x^0 \\ v(a,y) &= v(-a,y) \\ u(x,b) &= u(x,-b) \\ v(x,b) &= v(x,-b) + 2b\varepsilon_y^0 \\ w(x,y,z) &= \varepsilon_z^0 z \end{aligned} \quad (1)$$

and the traction continuity conditions are given as

$$\begin{aligned} \sigma_x(a,y) &= \sigma_y(-a,y) \\ \tau_{xy}(a,y) &= \tau_{xy}(-a,y) \\ \sigma_y(x,b) &= \sigma_y(x,-b) \\ \tau_{xy}(x,b) &= \tau_{xy}(x,-b) \end{aligned} \quad (2)$$

The uni-symmetric debonding problem can be solved by half of the RVE cell using the following boundary conditions.

$$y=0 \begin{cases} \tau_{xy}(x,0)=0 \\ \upsilon(x,0)=0 \end{cases} ; \; y=b \begin{cases} \tau_{xy}(x,b)=0 \\ \upsilon(x,b)=b\varepsilon_y^0 \end{cases} \quad (3)$$

For the doubly-symmetric debonding the symmetric characteristics holds about both the and axis. The overall problem can be modeled by a quarter cell of the RVE using the following boundary conditions

$$x=0 \begin{cases} u(0,y)=0 \\ \tau_{xy}(0,y)=0 \end{cases} ; \; x=a \begin{cases} u(a,y)=a\varepsilon_x^0 \\ \tau_{xy}(a,y)=0 \end{cases} \quad (4)$$

For purpose of determination of composite Young's moduli and poison's ratios through the composite constitutive relations $\bar{\sigma}_i = C_{ij}\bar{\varepsilon}_j \; (i,j=1,2,3 \text{ or } x,y,z)$, three district deformation states $(\varepsilon_x^0, \varepsilon_y^0, \varepsilon_z^0)$ are considered separately. The first two states are described by plane strain condition $(\varepsilon_z^0 = 0)$ with loadings $\varepsilon_x^0, \varepsilon_y^0 = 1$, 0 and 0,1 respectively. The third state represents the state of $\varepsilon_z^0 = 1$ with the cell boundary constrained. In the case of the cell under plane strain deformation $(w=0)$ with $\varepsilon_x^0 \neq 0, \varepsilon_y^0 = 0$, the average stresses with the help of Gauss theorem and traction continuity conditions are obtained as

$$\bar{\sigma}_x = \frac{1}{4ab}\int_s \sigma_x dS = \frac{1}{4ab}\oint xT_x ds = \begin{cases} \frac{1}{2b}\int_{-b}^{b} \sigma_x(a,y)dy & \text{uni-symmetric} \\ \frac{1}{b}\int_{0}^{b} \sigma_x(a,y)dy & \text{doubly-symmetric} \end{cases}$$

$$\bar{\sigma}_y = \frac{1}{4ab}\int_s \sigma_y dS = \frac{1}{4ab}\oint yT_y ds = \begin{cases} \frac{1}{2a}\int_{-a}^{a} \sigma_y(x,b)dx & \text{uni-symmetric} \\ \frac{1}{a}\int_{0}^{a} \sigma_y(x,b)dy & \text{doubly-symmetric} \end{cases}$$

$$\bar{\sigma}_z = \frac{1}{4ab}\int_S \sigma_z \, ds$$

where S is the area of the RVE. Due to the debonded interfaces, the line integral should be calculated along the boundary of the RVE as well as the interface between the fiber and matrix.

The composite elastic coefficient are defined as

$$\begin{aligned} &\bar{\sigma}_x = C_{11}\varepsilon_x^0, && \bar{\sigma}_y = C_{21}\varepsilon_x^0, && \bar{\sigma}_z = C_{31}\varepsilon_x^0 \\ &\bar{\sigma}_x = C_{12}\varepsilon_y^0, && \bar{\sigma}_y = C_{22}\varepsilon_y^0, && \bar{\sigma}_z = C_{32}\varepsilon_y^0 \\ &\bar{\sigma}_x = C_{13}\varepsilon_z^0, && \bar{\sigma}_y = C_{23}\varepsilon_z^0, && \bar{\sigma}_z = C_{33}\varepsilon_z^0 \end{aligned}$$

For the doubly symmetric debonding, $\tau_{xy}(a,y)=0$. However, for the uni-symmetric debonding $\tau_{xy}(a,y) \neq 0,$ the average shear stress is equal to zero due to the symmetry of τ_{xy} with respect to the x axis. Therefore, $C_{61}=C_{62}=C_{63}=0.$

In plane composite shear modulus:

For the composite under shear, straight cell boundaries may not remain straight after the composite has been deformed. Since the boundary displacements and tractions on any RVE must be compatible with those on the neighboring RVE, for the finite element modeling, constraints are imposed on the displacement at the edges. The displacement constraint boundary conditions used in obtaining the in-plane shear moduli for the RVE are given by

$$u(a,y)=u(-a,y)$$
$$v(a,y)=v(-a,y)+a\gamma_{xy}^{0}$$
$$u(x,b)=u(x,-b)+b\gamma_{xy}^{0}$$
$$v(x,b)=v(x,-b)$$

Since the in-plane shear is a plane strain problem, $w(x,y)=0$ over the whole RVE.

For the uni-symmetric debonding, the problem has symmetric geometry and skew symmetric loadings about the x axis. Thus the strain components must be skew-symmetric about the x axis.

$$\varepsilon_x(x,y)=-\varepsilon_x(x,y)$$
$$\varepsilon_y(x,y)=-\varepsilon_y(x,-y)$$
$$\gamma_{xy}(x,y)=\gamma_{xy}(x,-y)$$

The uni-symmetric debonding case can be analyzed by a half cell using the following conditions.

$$y=0\begin{cases}u(x,0)=0\\ \sigma_y(x,0)=0\end{cases};\quad y=b\begin{cases}u(x,b)=\frac{1}{2}b\gamma_{xy}^{0}\\ \sigma_y(x,b)=0\end{cases}$$

For the doubly symmetric debonding, the problem function exhibit the symmetric geometry and antisymmetric loading about the y axis. The boundary conditions for doubly-symmetric debonding are

$$x=0\begin{cases}\sigma_x(0,y)=0\\ v(0,y)=0\end{cases};\quad x=a\begin{cases}\sigma_x(a,y)=0\\ v(a,y)=\frac{1}{2}a\gamma_{xy}^{0}\end{cases}$$

The composite in plane shear modulus is defined as $\bar{\tau}_{xy}=G_{12}\bar{\gamma}_{xy}=G_{12}\gamma_{xy}^{0}=C_{66}\gamma_{xy}^{0}$

Due to anti-symmetric nature of the stress distributions, ν is either ν_m or ν_f depending on the region where the integrals are covered. Therefore $C_{16}=C_{26}=C_{36}=0$

The RVE does not have linear deformation along its edge.

RESULTS AND DISCUSSION

The ANSYS finite element package is used to analyze the RVE of the periodic rectangular model. The problems are modeled as a half cell or quarter cell for the uni-symmetric and doubly-symmetric debonding cases respectively. Four node plane element is used in the modeling. In order to illustrate the effect of interfacial debonding on the composite moduli, a brittle Carbon/Carbon ceramic matrix composite with fiber volume fraction 40% is studied. The fiber and matrix are assumed to be isotropic with modulus of elasticity 200 GPa and modulus of rigidity 70 GPa.

The effect of debonding angle on the nine composite moduli constants for uni-symmetric debonding case with the x, y and z structural axes shown in fig.1 coinciding with the material principal axes 1,2 and 3 as shown in fig.2-4. The moduli of composite are presented in terms of engineering constants $E_1, E_2, \ldots$ etc. In the extreme cases of perfect bonding, the composite with a square periodic array has only six independent elastic constants i.e., $E_1=E_2, G_{13}=G_{23}, \nu_{13}=\nu_{23}$. The axial modulus E_3 is insensitive to the debonded interface conditions. The numerical values of E_3 decreases from 133.02 GPa for perfect bonding case to 132.95 GPa for 90° debonding. For engineering applications, the axial modulus can be accurately predicted from the rule of mixtures regardless of the bonding between the fiber and the matrix.

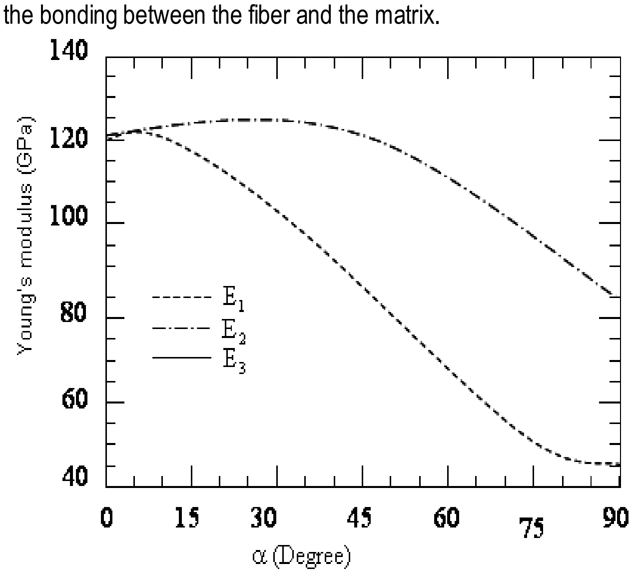

Fig.2. The effect of debonding angle on the composite Young's moduli for uni-symmetric debonding.

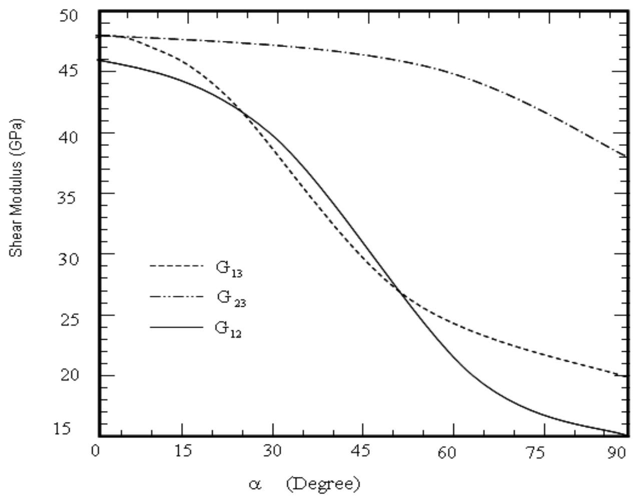

Fig.3. The effect of debonding angle on the composite shear moduli for uni-symmetric debonding.

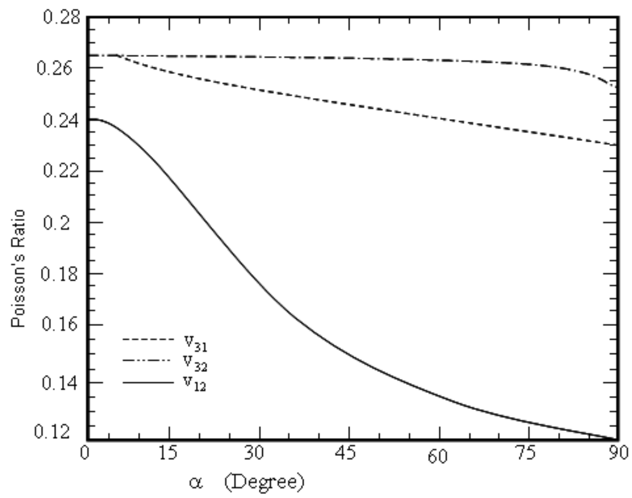

Fig.4. The effect of debonding angle on the composite Poisson's ratio for uni-symmetric debonding.

Fig.5-7 illustrates the effect of debonding angle on the composite moduli for doubly-symmetric debonding. When $\alpha = 0°$ which is perfectly bonded case, the problem is identical to the case of uni-symmetric debonding with $\alpha = 0°$. The $\alpha = 90°$ case also presents identical properties along x and y axis, hence it has negligible effect on the axial modulus E_3 for doubly-symmetric debonding. Also from fig. 2-7, E_2, G_{23}, ν_{32} and ν_{31} are affected insignificantly by the debonding; E_1, G_{13}, G_{12} and ν_{12} are influenced strongly by the debonding at the fiber-matrix boundary.

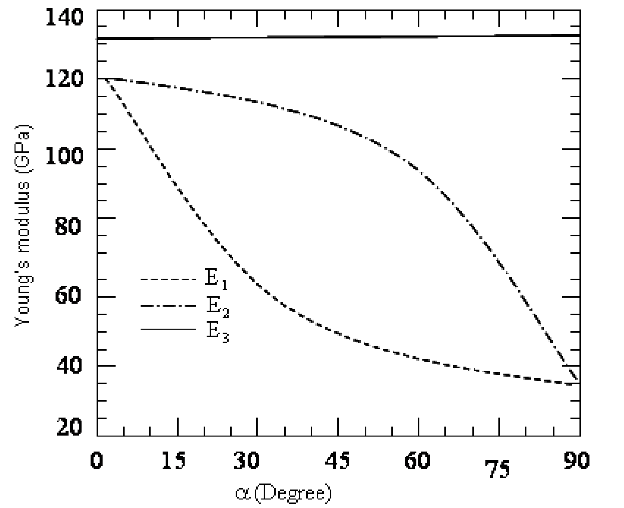

Fig.5. The effect of debonding angle on the composite Young's moduli for doubly-symmetric debonding.

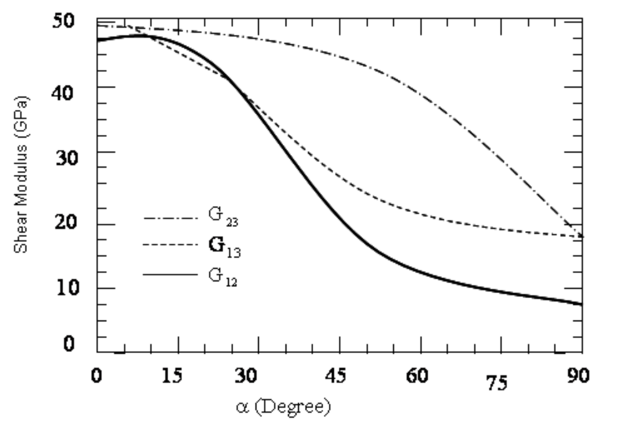

Fig.6. The effect of debonding angle on the composite shear moduli for doubly- symmetric debonding.

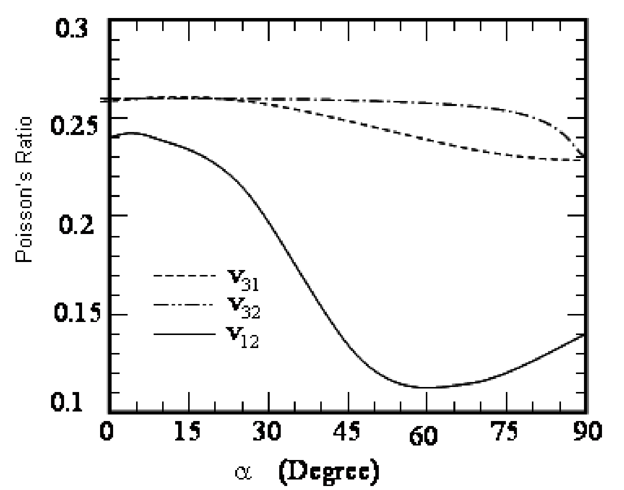

Fig.7. The effect of debonding angle on the composite Poisson's ratio for doubly- symmetric debonding.

Parametric studies are conducted for shear moduli of the composite shown in fig 8-10. In these studies the values of the ratios G_f / G_m varying from 1 to 1000 and five fiber volume fractions covering from 40% to 75% are considered. In calculating G_{12}, the Poisson's ratios of the fiber and the matrix are assumed to be 0.3 and 0.222 respectively. The effect of the ratios G_f / G_m on longitudinal shear modulus G_{23} and in-plane shear moduli G_{12} for a perfectly bonded composite shown in fig.9. For perfect bonding $G_{13} = G_{23}$. These figures show that the composite moduli approach asymptotic values as the values of G_f / G_m increase.

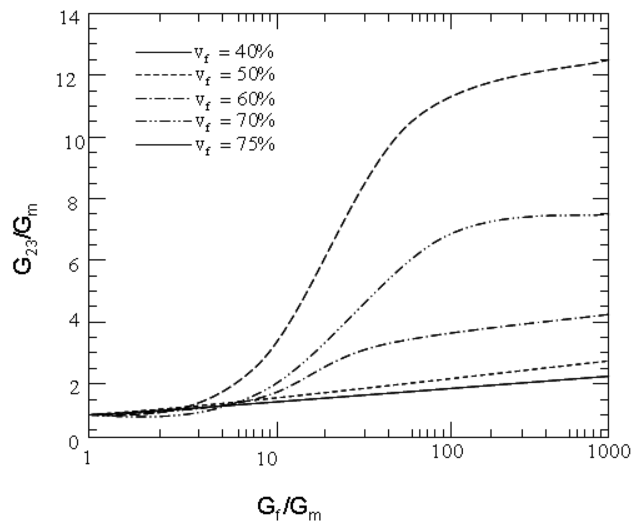

Fig.8. Effect of fiber volume fraction on the composite shear moduli G23 for perfectly bonded interface.

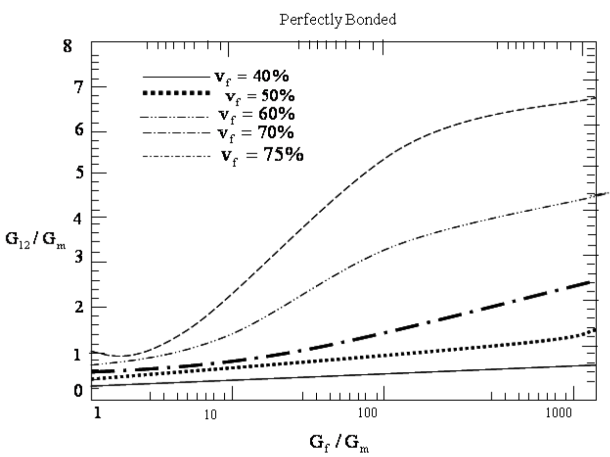

Fig.9. Effect of fiber volume fraction on the composite shear moduli for perfectly bonded interface. 12 G

Fig.10 depict the variations of composite shear modulus and G_f / G_m for $45°$ uni-symmetric debonded composite. It can be seen that for this case the degree of debonding causes little change in G_{23}, but it does reduces the other two module G_{13} and G_{23} significantly.

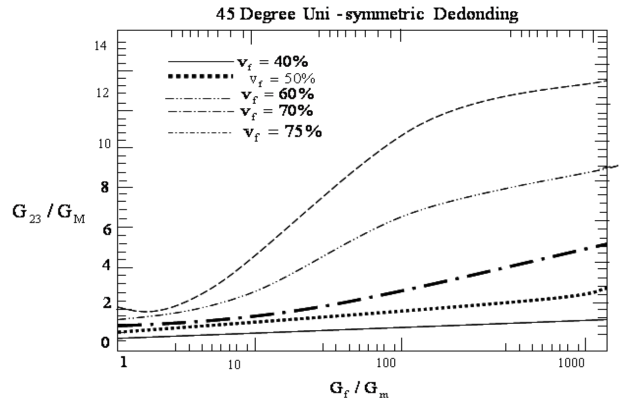

Fig.10. Effect of fiber volume fraction on the composite shear moduli G_{23} for uni-symmetrically 45° bonded interface.

The trend is some what reversed for G_{13}. The in-plane composite shear moduli are lower than the shear modulus of the matrix in the range of fiber volume fractions. The increase of shear modulus of the fiber still causes the increase in the composite modulus, while the increase in the volume fraction of the fiber will decrease the overall modulus of the composite. This phenomena can be interpreted by the fact that the in-plane shear loading is carried primarily by the matrix rather than the fiber for doubly-symmetric debonding composite.

CONCLUSIONS

The effect of interfacial debonding on the composite moduli is studied using two dimensional finite element methods. The traction and displacement compatability conditions along the boundaries of RVE are imposed as the boundary conditions in the finite element analysis. Nine independent material constants are required to be investigated in assessing the degradation of the composite moduli due to debonding. Further, only a half cell or quarter cell need to be modeled for uni-symmetric and doubly-symmetric debonding. Since the edge of the RVE does not remain straight under shear, a new definition of composite shear strains based on surface averaging is thus proposed. The degree of debonding has very little effect on the axial composite modulus E_3.The transverse Young's modulus E_1 and longitudinal shear modulus G_{13}, in-plane shear modulus G_{12}, and poison's ration ν_{12} are affected significantly by the degree of the debonding. This phenomena is mainly due to the lack of stress transfer across the debonded surfaces from the loading.

The composite shear moduli approach an asymptote as the ratios of G_f / G_m are very large for all the cases.

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